Pricing and hedging a European put

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For a desk short 100 put contracts, I would rebalance when delta moves by 2.5 points. In the base simulation this rule averaged 84 trades per path, versus 249 with daily hedging. Trading cost fell by $212 while the mean loss in the worst 5% of paths increased by $32 over the year.

The reference is a one-year put on a $36 stock with a $40 strike, 20% volatility, a 6% continuously compounded interest rate and zero dividends. I use 100 units per contract, making the book equivalent to 10,000 puts. Each option is cash-settled at expiry.[1]

One year of hedging the same short position

Hedge rule	Trades	Trading cost	Mean P&L	Worst 5% mean P&L
Daily	249	$1,132	−$1,149	−$5,477
2.5-point band	84	$920	−$930	−$5,509
5-point band	39	$718	−$724	−$6,355
10-point band	16	$526	−$527	−$9,288
Every 5 sessions	51	$621	−$612	−$9,402

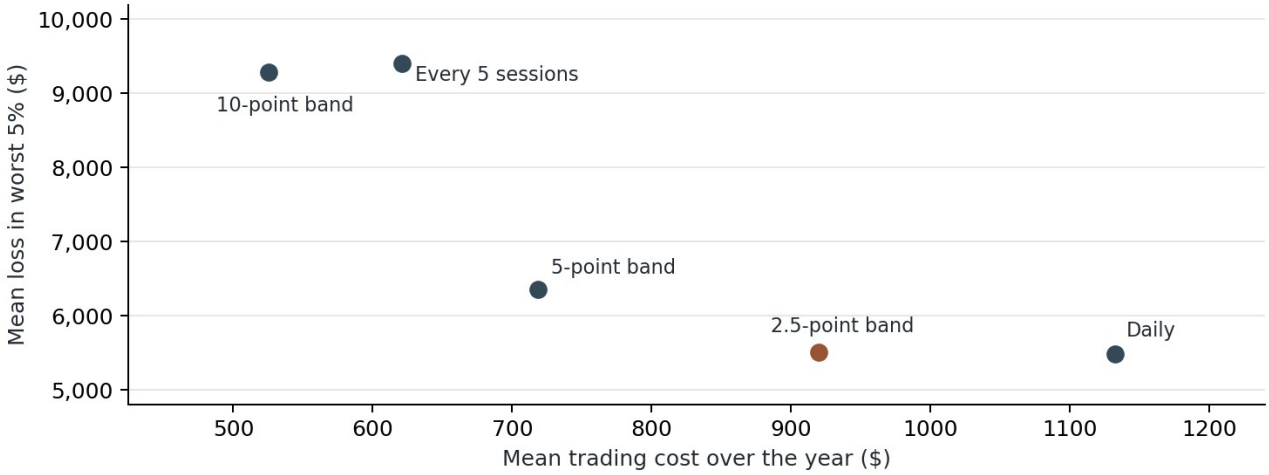
A delta point is 0.01 share per put. The chosen trigger is a gap of 250 shares between the desired hedge and the shares held. On a breach, the desk trades back to the full theoretical hedge. The comparison charges 5 bp per dollar of stock bought or sold, including entry and liquidation.[2]

1 QuantLib, EquityOption European contract example · George Dharma, Pricing and hedging a European put, accompanying research files, contract and execution assumptions.

2 George Dharma, Pricing and hedging a European put, accompanying research files, Band outcomes, Paired effects and Cash ledger.

Small savings can come with a larger tail loss

The five-point band saves another $201 in average execution cost compared with the 2.5-point trigger. In the same base case, its worst 5% of outcomes lose a further $846 on average. A weekly schedule produces a similar mean trading cost with a considerably larger tail loss.



What happens when the pricing assumptions fail

Scenario, worst 5% mean P&L	Daily	2.5-point band	5-point band
20% realised volatility	−$5,477	−$5,509	−$6,355
30% realised volatility	−$35,212	−$34,979	−$34,990
20% diffusion plus jumps	−$57,433	−$57,059	−$56,547

All hedge rules still use 20% implied volatility. When the simulated stock moves at 30%, the option premium leaves the seller exposed to the extra movement. When jumps are possible, a price gap can occur between two hedge trades. Shortening a trading interval cannot remove that loss.[3]

3 George Dharma, Pricing and hedging a European put, accompanying research files, Band outcomes, Paired effects and Cash ledger.

From an option price to cash at expiry

The Black–Scholes value is $3.84431 per put, or $38,443 for the short book. Delta is −0.55045 share per long put. Selling the puts therefore calls for an initial short hedge of about 5,505 shares. At expiry the model closes the stock trade and pays the buyer any amount by which the strike exceeds the stock price.

Put value per share by stock price and implied volatility

Stock price	15% volatility	20% volatility	25% volatility
$32	$6.04	$6.48	$7.01
$34	$4.46	$5.06	$5.69
$36	$3.14	**$3.84**	$4.56
$38	$2.10	$2.85	$3.60
$40	$1.34	$2.07	$2.81

The outlined cell marks the reference premium. Each column changes implied volatility at inception, holding the strike, time, interest rate and dividend yield fixed.[4]

Cross-check the premium

A 3,200-step binomial tree prices the put at $3.84418. One million antithetic Monte Carlo paths give $3.84526 with a 95% simulation interval of $3.84094–$3.84957. A hypothetical $3.80 bid and $3.90 ask imply volatilities of 19.69% and 20.39%. These are example quotes applied to the same contract.[5]

Measure the benefit on matched paths

Each stress uses 30,000 paths and 252 trading sessions. All five hedge rules see the same price path within a case. In the base case the 2.5-point rule improves mean P&L by $219 relative to daily trading. The paired 95% simulation interval is $209–$229. It measures sampling uncertainty under the specified model.

The cash account earns or pays 6%. Stock can be borrowed without an extra fee, and each trade pays the same spread. That keeps the cost of changing hedge rules visible. A desk facing stock-borrow charges or margin calls would need to add those cash flows before using the dollar estimates.[6]

4 George Dharma, Pricing and hedging a European put, accompanying research files, Pricing desk.

5 QuantLib, EquityOption European contract example · George Dharma, Pricing and hedging a European put, accompanying research files, Price methods, Monte Carlo and illustrative quotes.

6 George Dharma, Pricing and hedging a European put, accompanying research files, Band outcomes, Paired effects and Cash ledger.